

## Review of Algebra and Precalculus topics for AP Calculus AB

### Summer 2010

In the first three days of the 2010-2011 school year we will quickly cover the first chapter of the AP Calculus AB textbook, which provides a review of topics covered in precalculus that are essential for the study of calculus. This packet is designed to help you keep that information fresh so that the review is quick and painless. This will allow us to begin the study of calculus much more quickly, and therefore give us more time to ease your understanding of the material. Please read through one section each week, doing the problems as you go through it. Answers are provided at the back of the packet.

A group for this class has been created on [www.facebook.com](http://www.facebook.com). This will provide me to easily communicate with students and, more importantly, them to communicate with each other. This group is private, which means that nobody who is not a member of the group can view anything in the group, including group members or discussion threads. Membership is also limited to those who receive an invitation. Any member of the class can receive an invitation to the group by sending me an email address that is associated with a Facebook account. I will immediately destroy any record of email addresses as soon as an invitation is sent.

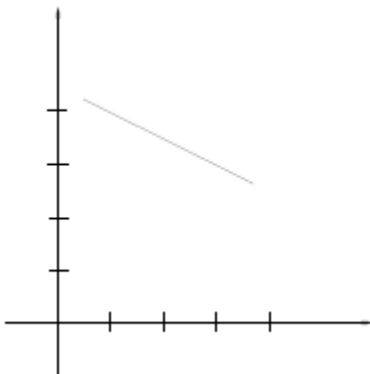
If your parents have any question about registration or the web site, they may email me ([kadams@limestone.k12.il.us](mailto:kadams@limestone.k12.il.us)) or call me at LCHS (697-9742).

This web site will be used for messages that pertain to the entire group. A calendar will be kept concerning when assignments are made, when tests are scheduled, and important dates on the road to the AP Calculus AB exam. Lesson plans will also be kept on this site, allowing you to see my teaching notes.

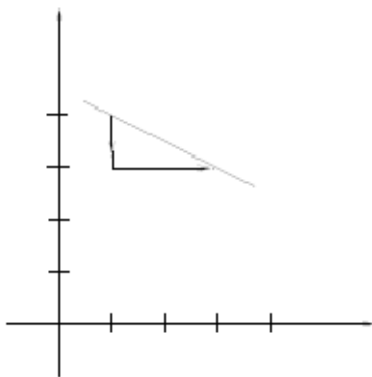
### Section 1: Slope

Recall that slope  $m$  is defined as  $\frac{\text{rise}}{\text{run}}$  or  $\frac{\text{change in } y}{\text{change in } x}$ . Rise is the distance traveled vertically and is positive if traveling up (in the positive  $y$  direction). Run is the distance traveled horizontally and is positive if traveling right (in the positive  $x$  direction). We estimate the slope of a line by drawing horizontal and vertical lines and estimating rise and run.

EXAMPLE: Approximate the slope of the given line.



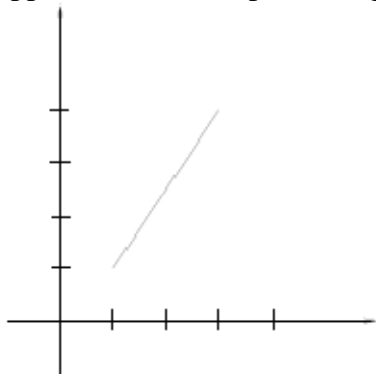
SOLUTION: We draw in vertical and horizontal lines and estimate the rise as -1 and the run as 2. This gives an approximate slope of  $-\frac{1}{2}$ .



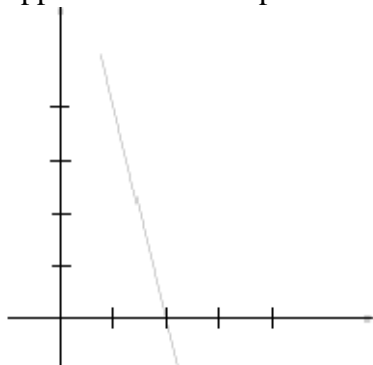
Note: If we had interpreted the rise as 1 and the run as -2, we would get the same answer of  $-\frac{1}{2}$ .

Try the following problems. The answers are in the solution section.

1. Approximate the slope of the given line.



2. Approximate the slope of the given line.



### Section 2: Lines

A line which goes through points  $(x_1, y_1)$  and  $(x_2, y_2)$  has slope  $m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$ . If a line has

slope  $m$  and goes through point  $(x_1, y_1)$ , we call an arbitrary point on the line  $(x, y)$  and get that  $m = \frac{y - y_1}{x - x_1}$ .

Rewriting, we get the usual form  $y - y_1 = m(x - x_1)$ .

EXAMPLE: Find the line through points  $(2,-3)$  and  $(1,4)$ :

The slope is  $m = \frac{4 - (-3)}{1 - 2} = \frac{7}{-1} = -7$ . To find the equation, we can use either point. Using the point  $(2,-3)$ , we get  $y - (-3) = -7(x - 2)$ .

The equation of the line with slope  $m$  and  $y$ -intercept  $b$  is  $y - b = m(x - 0)$  or  $y = mx + b$ . (Recall: Because points on the  $y$ -axis have  $x = 0$ , a  $y$ -intercept of  $b$  indicates that point  $(0,b)$  is on the line.)

Thus, the above equation in the example can be rewritten as  $y = -7x + 11$ . If we use the point  $(1,4)$ , we get  $y - 4 = -7(x - 1)$  or  $y = -7x + 11$ .

Try the following problems. Recall that parallel lines have the same slopes. The answers are in the solutions section.

1. Find the equation of the line through points  $(4,-2)$  and  $(-1,3)$ .
2. Find the equation of the line with  $x$ -intercept 3 and  $y$ -intercept -2.
3. Find the equation of the line that goes through the point  $(2,3)$  and is parallel to the line  $2x + 6y = 1$

### Section 3: Exponents

When you get confused about the rules for exponents, think about what the exponent means.

EXAMPLE:  $x^4 x^2 = (xxxx)(xx) = xxxxxx = x^6$ . The exponents are added.

EXAMPLE:  $(x^4)^2 = (x^4)(x^4) = (xxxx)(xxxx) = xxxxxxxx = x^8$ . The exponents are multiplied.

EXAMPLE:  $\frac{x^5}{x^2} = \frac{xxxxx}{xx} = \left(\frac{x}{x}\right)\left(\frac{x}{x}\right)xxx = (1)(1)xxx = x^3$ . The exponents are subtracted.

EXAMPLE:  $(xy)^2 = (xy)(xy) = xxyy = x^2 y^2$ . In general,  $(xy)^n = x^n y^n$ .

BEWARE:  $(x + y)^2 \neq x^2 + y^2$  because  $(x + y)^2 = (x + y)(x + y) = x^2 + xy + xy + y^2 = x^2 + 2xy + y^2$ .

Summary of Rules of Exponents:

1.  $x^n x^m = x^{n+m}$
2.  $(x^n)^m = x^{nm}$
3.  $\frac{x^n}{x^m} = x^{n-m}$
4.  $(xy)^n = x^n y^n$

EXAMPLE:  $\frac{3x^4(xy)^2}{(2x^2)^3} = \frac{3x^4 x^2 y^2}{2^3 (x^2)^3} = \frac{3x^6 y^2}{8x^6} = \frac{3x^{6-6} y^2}{8} = \frac{3y^2}{8}$

Try the following problems, simplifying completely and writing answers with no negative exponents. The answers are in the solution section.

- $\frac{(3x^2y^3)^2 x^3}{4xy^7}$
- $\left(\frac{2x}{y^2}\right)^3 (xy^3)^2$

#### Section 4: Negative and Fractional Exponents

We will use the rules of exponents in the above section to understand what negative and fractional exponents mean.

- $x^0 = 1$       Why?  $\frac{x^2}{x^2} = x^{2-2} = x^0$  and  $\frac{x^2}{x^2} = 1$ .
- $x^{-n} = \frac{1}{x^n}$       Why? Because  $x^n x^{-n} = x^{n+(-n)} = x^0 = 1$ , we know  $x^n x^{-n} = 1$ . When we divide both sides by  $x^n$ , we get  $x^{-n} = \frac{1}{x^n}$ .
- $x^{\frac{1}{n}} = \sqrt[n]{x}$       Why?  $\left(x^{\frac{1}{n}}\right)^n = x^{\frac{n}{n}} = x^1 = x$ . Thus,  $\left(x^{\frac{1}{n}}\right)^n = x$  and we take the  $n$ th root of both sides to get  $x^{\frac{1}{n}} = \sqrt[n]{x}$ .
- $x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$       Why?  $x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$  and  $x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$ .

EXAMPLE: Write  $x^{-\frac{2}{3}}$  without fractions or negatives in the exponent:  $x^{-\frac{2}{3}} = \frac{1}{x^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{x^2}}$ . Another correct

answer would be  $\frac{1}{\left(\sqrt[3]{x}\right)^2}$ .

EXAMPLE: Write  $\frac{1}{x^2}$  in the form  $x^a$ :  $\frac{1}{x^2} = x^{-2}$ .

EXAMPLE: Write  $\frac{1}{\sqrt[5]{x^3}}$  in the form  $x^a$ :  $\frac{1}{\sqrt[5]{x^3}} = \frac{1}{\left(x^3\right)^{\frac{1}{5}}} = \frac{1}{x^{\frac{3}{5}}} = x^{-\frac{3}{5}}$ .

Try the following problems. The answers are in the solution section.

- Write  $x^{\frac{3}{4}}$  without fractions or negatives in the exponent.
- Write  $x^{-\frac{3}{2}}$  without fractions or negatives in the exponent.
- Write  $(\sqrt{x})^5$  in the form  $x^a$ .
- Write  $\frac{1}{\sqrt[3]{x^7}}$  in the form  $x^a$ .

## Section 5: Logarithms

The inverse function of the exponential function is the logarithmic function. For  $f(x) = 10^x$ , then  $f^{-1}(x) = \log_{10} x = \log x$ , the common logarithmic function. For  $g(x) = e^x$ , then  $g^{-1}(x) = \log_e x = \ln x$ , the natural logarithmic function. Natural logarithms are more common than logarithms of other bases in the study of the calculus. Logarithms are only defined when the argument of the logarithm is greater than zero.

Some basic properties of logarithms of any base (illustrated using the natural logarithmic function) are:

1.  $\ln 1 = 0$
2.  $\ln e^k = k$  for every real number  $k$
3.  $e^{\ln v} = v$  for every  $v > 0$
4. For all  $k$  and  $v, w > 0$ ,
  - a.  $\ln(vw) = \ln v + \ln w$
  - b.  $\ln\left(\frac{v}{w}\right) = \ln v - \ln w$
  - c.  $\ln v^k = k \cdot \ln v$

EXAMPLE: Given that  $\ln 7 \approx 1.9459$  and  $\ln 9 \approx 2.1972$ , then  $\ln 63 = \ln 7 + \ln 9 \approx 1.9459 + 2.1972 = 4.1431$

EXAMPLE: Given that  $\ln 18 \approx 2.8904$  and  $\ln 6 \approx 1.7918$ , then  $\ln 3 = \ln 18 - \ln 6 \approx 2.8904 - 1.7918 = 1.0986$

EXAMPLE: Given that  $\ln 50 \approx 3.9120$ , then  $\ln \sqrt[3]{50} = \frac{1}{3} \ln 50 \approx \frac{1}{3}(3.9120) = 1.3040$

EXAMPLE: Write  $\ln 3x + 4 \ln x - \ln 3xy$  as a single logarithm. Because  $4 \ln x = \ln x^4$ , we have

$$\ln 3x + \ln x^4 - \ln 3xy = \ln \frac{3x(x^4)}{3xy} = \ln \frac{x^4}{y}$$

Simplify the following expressions. The answers are in the solution section.

1.  $\ln x^2 + 3 \ln y$
2.  $2 \ln x - 3(\ln x^2 + \ln x)$
3.  $\ln(e^2 x) + \ln(ey) - 3$

## Section 6: Fractions

We will look at some numerical examples first. For each example, try it yourself before you look at the solution.

EXAMPLE:  $\frac{2}{5} \cdot \frac{3}{7}$

Answer:  $\frac{(2)(3)}{(5)(7)} = \frac{6}{35}$

EXAMPLE:  $\frac{\frac{2}{5}}{\frac{3}{7}}$

Answer:  $\frac{2}{5} \cdot \frac{7}{3} = \frac{14}{15}$

EXAMPLE:  $\frac{2}{5} + \frac{3}{7}$

Answer:  $\frac{2}{5} \cdot \frac{7}{7} + \frac{3}{7} \cdot \frac{5}{5} = \frac{14}{35} + \frac{15}{35} = \frac{29}{35}$

Now let's try some examples with variables. Try yourself before looking at answers.

EXAMPLE:  $\frac{x}{y} \cdot z$

Answer:  $\frac{x}{y} \cdot \frac{z}{1} = \frac{xz}{y}$

EXAMPLE:  $\frac{\frac{x}{y}}{\frac{z}{y}}$

Answer:  $\frac{x}{y} \cdot \frac{y}{z} = \frac{xy}{yz} = \frac{x}{z}$

EXAMPLE:  $\frac{x}{y} - \frac{y}{z}$

Answer:  $\frac{x}{y} \cdot \frac{z}{z} - \frac{y}{z} \cdot \frac{y}{y} = \frac{xz - y^2}{yz}$

EXAMPLE:  $\frac{2}{x+1} - \frac{3}{x-2}$

Answer: 
$$\begin{aligned} \frac{2}{x+1} \cdot \frac{x-2}{x-2} - \frac{3}{x-2} \cdot \frac{x+1}{x+1} &= \frac{2(x-2) - 3(x+1)}{(x+1)(x-2)} \\ &= \frac{2x-4-3x-3}{x^2-x-2} \\ &= \frac{-x-7}{x^2-x-2} \end{aligned}$$

Try the following problems. The answers are in the solution section.

1.  $\frac{\frac{x^2-x}{y}}{\frac{x}{y}}$
2.  $\frac{x+1}{x-2} - \frac{x+3}{x-4}$

### Section 7: Functions

For the function  $f$  defined by  $f(x) = 3x - x^2$ , we express in English that the function takes an input, multiplies it by 3, and subtracts its square. So  $f(-2) = 3(-2) - (-2)^2 = -10$  and

$$f(x+h) = 3(x+h) - (x+h)^2 = 3x+3h - (x^2+2xh+h^2) = 3x+3h - x^2 - 2xh + h^2.$$

For the function  $g$  defined by  $g(x) = 1 - 3x^2$ , we express in English that the function takes an input, squares it, multiplies the result by 3 and subtracts that result from 1. So  $g(-2) = 1 - 3(-2)^2 = -11$  and

$$g(x+h) = 1 - 3(x+h)^2 = 1 - 3(x^2+2xh+h^2) = 1 - 3x^2 - 6xh + 3h^2.$$

Here are some for you to try. The answers are in the solution section.

1. If  $f(x) = 2x - x^2$ , find  $f(x+h)$  and simplify completely.
2. If  $f(x) = \frac{x+1}{2x-1}$ , find  $f(x+h)$  and simplify completely.
3. If  $f(x) = 3 - 2x^2$ , find  $f(x+h)$  and simplify completely.

## Section 8: Quadratic Equations

We can solve quadratic equations by factoring or by using the quadratic formula.

**EXAMPLE:** Solve  $x^2 - x = 6$  by factoring: We put all terms on one side, getting  $x^2 - x - 6 = 0$ . Next, factor to get  $(x - 3)(x + 2) = 0$ . If a product  $ab$  equals zero, then either  $a = 0$  or  $b = 0$ . For our problem, we conclude  $x - 3 = 0$  or  $x + 2 = 0$ . The answer is  $x = 3$  or  $x = -2$ .

Notice that if  $ab = 6$ , we cannot conclude  $a = 6$  or  $b = 6$ . Maybe  $a = 2$  and  $b = 3$  or  $a = \frac{1}{2}$  and  $b = 12$ . So, if you factored without moving all terms to one side to get  $x(x - 1) = 6$ , you cannot conclude that  $x = 6$  or  $x - 1 = 6$ .

**EXAMPLE:** Solve  $x^2 - x = 6$  by using the quadratic formula: Again, we need to put all terms on one side, getting  $x^2 - x - 6 = 0$ . To solve  $ax^2 + bx + c = 0$ , the quadratic formula tells us that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

In this example,  $a = 1$ ,  $b = -1$ , and  $c = -6$ , so  $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-6)}}{2(1)} = \frac{1 \pm \sqrt{25}}{2} = \frac{1 \pm 5}{2}$ . The solution is

$$x = \frac{1+5}{2} = 3 \text{ or } x = \frac{1-5}{2} = -2.$$

Here are some problems to try. The answers are in the solution section.

1. Solve the equation for  $x$ :  $x^2 - 2x = 8$ .

2. Solve the equation for  $x$ :  $\frac{2}{x} + 1 = x + 2$ .

## Section 9: Common Mistakes

Remember that  $\sqrt{x^2 + y^2} \neq x + y$ . For example  $\sqrt{2^2 + 3^2} \neq 2 + 3$  because  $\sqrt{2^2 + 3^2} = \sqrt{13}$ . Similarly,

$\sqrt{x^2 - y^2} \neq x - y$ . However, if  $x$  and  $y$  are nonnegative,  $\sqrt{x^2 y^2} = xy$  and  $\sqrt{\frac{x^2}{y^2}} = \frac{x}{y}$ .

In a similar vein,  $(x + y)^2 \neq x^2 + y^2$  because  $(x + y)^2 = (x + y)(x + y) = x^2 + 2xy + y^2$ . However,  $(xy)^2 = x^2 y^2$

because  $(xy)^2 = (xy)(xy) = xxyy = x^2 y^2$ . Also  $(x - y)^2 \neq x^2 - y^2$  but  $\left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2}$ .

Care needs to be taken when deciding if canceling is possible. We have that  $\frac{x + y}{x} \neq y$  but  $\frac{xy}{x} = y$  because

$$\frac{xy}{x} = \left(\frac{x}{x}\right)y = (1)y = y.$$

Recall that, for inequalities, if we multiply or divide both sides by a negative number, the inequality sign gets changed. We know that  $-3 \leq 2$  but if we multiply by  $-2$ , we get  $(-3)(-2) \geq 2(-2)$  because  $6 \geq -4$ .

EXAMPLE: Solve  $3 - 2x \leq 9$  for  $x$ : We get  $-2x \leq 9 - 3$  or  $-2x \leq 6$  or  $x \geq -3$ . The answer in interval notation is  $[-3, \infty)$ .

EXAMPLE: Solve  $x + 2 > 4x - 1$  for  $x$ : We get  $x - 4x < -1 - 2$  or  $-3x > -3$  or  $x < 1$ . The answer in interval notation is  $(-\infty, 1)$ .

Decide if the following statements are true or false. Correct those that are false. Assume that all variables are nonnegative. Answers are in the solution section.

1.  $\sqrt{16+9} = 4+3 = 7$

2. If  $3x > 12$ , then  $x > 4$ .

3.  $\frac{x+3y}{y} = x+3$

4.  $(2+3)^3 = 8+27$

5. If  $-2x < 4$ , then  $x \geq -2$ .

6.  $\sqrt[3]{\frac{x^3}{y^3}} = \frac{x}{y}$

7. If  $\frac{x}{-4} > 3$ , then  $x < -12$ .

8.  $\sqrt{x^2 - 9} = x - 3$

9.  $\sqrt{9x^2} = 3x$

10.  $\frac{xy+3z}{z} = xy+3$

11.  $(3x)^2 = 3x^2$

12.  $\frac{xy^2z}{yz} = xy$

13.  $(2y)^3 = 8y^3$

### Section 10: Trigonometry

Know and review the attached sheet. You should be able to pull any of the information on this sheet from memory.



Degree	Radian	sin	cos	tan	cot	sec	csc
0°	0	0	1	0	$\infty$	1	$\infty$
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	$\infty$	0	$\infty$	1

### Reference Angle Rules

Quadrant II: $t' = \pi - t$	Quadrant I: $t' = t$
Quadrant III: $t' = t - \pi$	Quadrant IV: $t' = 2\pi - t$

### Basic Trigonometric Identities

#### Reciprocal Identities

$$\sin x = \frac{1}{\csc x}$$

$$\cos x = \frac{1}{\sec x}$$

$$\tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

#### Quotient Identities

$$\frac{\sin x}{\cos x} = \tan x$$

$$\frac{\cos x}{\sin x} = \cot x$$

#### Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\tan^2 x = \sec^2 x - 1$$

$$1 = \csc^2 x - \cot^2 x$$

$$\cos^2 x = 1 - \sin^2 x$$

$$1 = \sec^2 x - \tan^2 x$$

$$\cot^2 x = \csc^2 x - 1$$

### Sum and Difference Identities

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

### Double-Angle and Half-Angle Identities

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$= 2 \cos^2 x - 1$$

$$= 1 - 2 \sin^2 x$$

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\begin{aligned} \tan \frac{x}{2} &= \frac{\sin x}{1 - \cos x} \\ &= \frac{1 + \cos x}{\sin x} \end{aligned}$$

## Section 11: Solutions

### Section 1:

$$1. m = \frac{\text{rise}}{\text{run}} = \frac{3}{2}$$

$$2. m = \frac{\text{rise}}{\text{run}} = \frac{-4}{1} = \frac{4}{-1} = -4$$

### Section 2:

$$1. m = \frac{3 - (-2)}{-1 - 4} = -1 \text{ and the equation of the line is } y + 2 = -1(x - 4).$$

$$2. \text{ The points are } (3,0) \text{ and } (0,-2). \text{ So } m = \frac{-2 - 0}{0 - 3} = \frac{2}{3} \text{ and the equation of the line is } y = \frac{2}{3}(x - 3).$$

$$3. \text{ To find the slope of the line } 2x + 6y = 1, \text{ we solve for } y: 6y = 1 - 2x \text{ so } y = \frac{1}{6} - \frac{2}{6}x. \text{ The slope is } -\frac{2}{6} = -\frac{1}{3} \text{ and the equation of the parallel line is } y - 3 = -\frac{1}{3}(x - 2).$$

### Section 3:

$$1. \frac{(3x^2y^3)^2 x^3}{4xy^7} = \frac{3^2(x^2)^2(y^3)^2 x^3}{4xy^7} = \frac{9x^4y^6x^3}{4xy^7} = \frac{9x^7y^6}{4xy^7} = \frac{9x^{7-1}y^{6-7}}{4} = \frac{9x^6y^{-1}}{4} = \frac{9x^6}{4y}$$

$$2. \left(\frac{2x}{y^2}\right)^3 (xy^3)^2 = \frac{2^3 x^3}{(y^2)^3} x^2 (y^3)^2 = \frac{8x^3 x^2 y^6}{y^6} = 8x^5$$

### Section 4:

$$1. x^{\frac{3}{4}} = (x^3)^{\frac{1}{4}} = \sqrt[4]{x^3} \dots \text{also } x^{\frac{3}{4}} = \left(x^{\frac{1}{4}}\right)^3 = (\sqrt[4]{x})^3$$

$$2. x^{-\frac{3}{2}} = \frac{1}{x^{\frac{3}{2}}} = \frac{1}{\sqrt{x^3}}$$

$$3. (\sqrt{x})^5 = \left(x^{\frac{1}{2}}\right)^5 = x^{\frac{5}{2}}$$

$$4. \frac{1}{\sqrt[3]{x^7}} = \frac{1}{\left(x^{\frac{1}{3}}\right)^7} = \frac{1}{x^{\frac{7}{3}}} = x^{-\frac{7}{3}}$$

### Section 5:

$$1. \ln x^2 + 3 \ln y = \ln x^2 + \ln y^3 = \ln(x^2 y^3)$$

$$2. 2 \ln x - 3(\ln x^2 + \ln x) = \ln x^2 - 3 \ln x^2 - 3 \ln x = -2 \ln x^2 - 3 \ln x = \ln(x^{-4}) - \ln x^3 = \ln\left(\frac{1}{x^4}\right) - \ln x^3 = \ln\left(\frac{1}{x^7}\right)$$

$$3. \ln(e^2 x) + \ln(ey) - 3 = \ln e^2 + \ln x + \ln e + \ln y - 3 = 2 \ln e + \ln x + \ln 3 + \ln y - 3 = \ln x + \ln y = \ln(xy)$$

## Section 6:

$$1. \frac{\frac{x^2 - x}{y}}{\frac{x}{y}} = \frac{x^2 - x}{y} \cdot \frac{y}{x} = \frac{x^2 - x}{x} = \frac{x(x-1)}{x} = x-1$$

$$2. \frac{x+1}{x-2} - \frac{x+3}{x-4} = \frac{x+1}{x-2} \cdot \frac{x-4}{x-4} - \frac{x+3}{x-4} \cdot \frac{x-2}{x-2} = \frac{x^2 - 3x - 4 - (x^2 + x - 6)}{x^2 - 6x + 8} = \frac{-4x + 2}{x^2 - 6x + 8}$$

## Section 7:

$$1. f(x+h) = 2(x+h) - (x+h)^2 = 2x+2h - (x^2 + 2xh + h^2) = 2x+2h - x^2 - 2xh - h^2$$

$$2. f(x+h) = \frac{x+h+1}{2(x+h)-1} = \frac{x+h+1}{2x+2h-1}$$

$$3. f(x+h) = 3 - 2(x+h)^2 = 3 - 2(x^2 + 2xh + h^2) = 3 - 2x^2 - 4xh - 2h^2$$

## Section 8:

1.  $x^2 - 2x - 8 = 0$ . Factoring, we get  $(x-4)(x+2) = 0$  so  $x-4 = 0$  or  $x+2 = 0$ . The answers are  $x = -4, 2$ .

2. Multiplying both sides of the equation by  $x$ , we get  $2 + x = x^2 + 2x$ . So  $0 = x^2 + x - 2$  and  $0 = (x+2)(x-1)$ . The answer is  $x = -2, 1$ .

## Section 9:

1. False:  $\sqrt{16+9} = \sqrt{25} = 5$

2. True

3. False:  $\frac{x+3y}{y} = \frac{x}{y} + 3$

4. False:  $(2+3)^3 = 5^3 = 125$

5. False: If  $-2x < 4$ , then  $x > -2$ .

6. True

7. True

8. False:  $\sqrt{x^2 - 9}$  cannot be simplified.

9. True

10. False:  $\frac{xy+3z}{z} = \frac{xy}{z} + 3$

11. False:  $(3x)^2 = 9x^2$

12. True

13. True